

On the Optimal Link Adaptation in Linear Relay Networks With Incremental Redundancy HARQ

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Abstract—In this letter, we investigate a joint power and rate optimization for a multihop relay network with incremental redundancy hybrid ARQ (HARQ-IR), where multiple relays are serially connected from a source to a destination. The optimization problem is to adjust both transmission rate and power of each node in order to maximize long-term average transmission rate (LATR) for given sum-power and delay constraints, which is unfortunately not mathematically tractable. Hence, we propose a sub-optimal algorithm to improve the LATR, which is computationally efficient and has comparable performance with the exhaustive search algorithm. Simulation results show that the proposed algorithm outperforms the conventional algorithms, including the HARQ-IR scheme with fixed power and rate and the Chase-combining HARQ scheme of which both power and rate are optimized for the same network.

Index Terms—Hybrid ARQ (HARQ), incremental redundancy, multi-hop relay network, rate adaptation, transmit power control.

I. INTRODUCTION

A MULTIHOP relay network where multiple relay nodes are serially connected through wireless channel from a source to a destination has been vastly investigated for wireless communications [1], [2], and it has also been adopted in wireless communication standards [3]. The multihop relay network model is applicable to mobile ad-hoc networks or vehicle-to-vehicle networks. The reliability of the multihop relay network can be improved by adopting Hybrid Automatic-Repeat-reQuest (HARQ) technique especially when wireless channels change fast and the transmitter cannot obtain the instantaneous channel gain [4].

There exist two types of HARQ technique: Chase-combining HARQ (HARQ-CC) [5] and incremental redundancy HARQ (HARQ-IR) [6]. The HARQ-IR has been known to yield the higher spectral efficiency than the HARQ-CC, but requires higher computation complexity than the HARQ-CC. Stanojev *et al.* investigated the optimal number of hops in the multihop relay network with the HARQ-CC [7]. Zhao and Valenti proposed a relay selection algorithm in the multihop relay network with both HARQ-CC and HARQ-IR [8]. However, we cannot select the relays to be used or change the number of hops since

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a routing protocol is given from network-layer in general. The rate selection scheme in the cooperative relay with HARQ-CC and HARQ-IR was considered in [9] but it is hard to be applied into the multi-hop relay network.

Recently, a joint rate and power optimization algorithm in the multihop relay network with the HARQ-CC was proposed without limitation on the number of retransmissions [10]. However, they did not consider the HARQ-IR for the multihop relay network even though it can further improve the long-term average transmission rate (LATR) in general. Furthermore, the optimization problem with HARQ-IR in the multihop relay network is totally different from the HARQ-CC. Therefore, in this letter, we investigate a joint power and rate optimization for a general M -hop relay network operating with the HARQ-IR, when total available transmit power of transmitters and time delay limitation, i.e., the maximum number of retransmissions, are given as constraints.

II. SYSTEM MODEL

We consider an M -hop relay network consisting of $M + 1$ nodes: a source (N_1), $M - 1$ relays (N_2, \dots, N_M), and a destination (N_{M+1}). We assume that the relays are serially located from N_1 to N_{M+1} in the order of the subscript of N . The signals received from other nodes except neighboring nodes are assumed to be negligible as in [10] and all nodes are assumed to have a single antenna. N_1 generates b bits of information message and this message is conveyed to N_{M+1} via half-duplex relays operating with a decode-and-forward manner. If N_m recovers the information message from N_{m-1} , then N_m re-encodes the message and forwards it to N_{m+1} . At each hop, the HARQ-IR is adopted.

In the m -th hop, N_m encodes an information message into codeword \mathbf{c}_m of length $T_m K$ from the codebook $\mathcal{C}_m \in \mathbb{C}^{T_m K}$ where K denotes the number of subblocks constituting a codeword and T_m denotes the number of symbols of a subblock. Each subblock is composed of different symbols and it can be correctly decoded if channel condition is good enough. Let us assume that N_m successfully decodes the message from N_{m-1} at the s -th time slot. Then, N_m transmits the first subblock of \mathbf{c}_m to N_{m+1} at the $(s + 1)$ -th time slot. If N_{m+1} successfully decodes the message at the $(s + 1)$ -th time slot, it also sends the first subblock of \mathbf{c}_{m+1} to N_{m+2} at the $(s + 2)$ -th time slot. On the other hand, if N_{m+1} does not decode the message from N_m at the $(s + 1)$ -th time slot, it sends negative acknowledgement (NACK) signal to N_m . We assume that the NACK signal is zero-delayed and error-free. If N_m receives a NACK signal from N_{m+1} , it transmits the second subblock of \mathbf{c}_m at the $(s + 2)$ -th time slot. This process is repeated until the destination, N_{M+1} , successfully decodes the message. The total number of time slots used to convey a message from N_1 to N_{M+1} is limited to L , which can be regarded as delay constraint. In

addition, the maximum number of retransmissions for each hop can be limited to L_m such that $\sum_{m=1}^M L_m = L$.¹ In the m -th hop, if a packet is not successfully decoded within L_m HARQ rounds, the packet is dropped. In this letter, we assume that L_m is identical for different m , i.e., $L_m = L/M$ and the sum of available transmit powers of all nodes is set to P^{total} .

Let $h_{m,k}$ denote the channel coefficient at the k -th HARQ round of the m -th hop. $h_{m,k}$ is assumed to be an independent, zero-mean complex Gaussian random variable with variance σ_m^2 , i.e., $h_{m,k} \sim \mathcal{CN}(0, \sigma_m^2)$. Let P_m denote the transmit power of N_m . We also assume an additive white Gaussian noise (AWGN) with variance, Ω , for all receivers. In this letter, we assume $\Omega = 1$ unless otherwise mentioned. We assume that transmitters only know the statistic of wireless channel but receivers know the channel exactly as in [10]–[12].

The transmission rate of the m -th hop at the first HARQ round is defined as $R_m \triangleq (b/T_m)$, $\forall m$ (b/s/Hz), which will be optimized later and R_m can be different for each m . Let S_m^i be the number of time slots used for transmitting the i -th message at the m -th hop. Then, the average transmission rate for D packets is given by $D b / (\sum_{i=1}^D \sum_{m=1}^M T_m S_m^i)$. If D goes to infinity, we obtain the long-term average transmission rate (LATR) as [10]

$$\mathcal{T} = \frac{1}{\sum_{m=1}^M \mathbb{E}[S_m]/R_m} \quad [\text{bps/Hz}], \quad (1)$$

where S_m denotes the random variable representing the number of transmissions in the m -th hop.

III. JOINT RATE AND POWER OPTIMIZATION

A. Problem Formulation

In the m -th hop, the outage probability after the k -th HARQ round for a given data rate, R_m , is given by [11] $p_{m,k}(R_m, P_m) = \Pr[\sum_{l=1}^k \log_2(1 + |h_{m,l}|^2 P_m) < R_m]$. The probability that a packet is successfully decoded after the k -th HARQ round in the m -th hop is denoted by

$$q_{m,k}(R_m, P_m) = p_{m,k-1}(R_m, P_m) - p_{m,k}(R_m, P_m).$$

In the m -th ($2 \leq m \leq M$) hop, the transmission happens only when a packet is successfully decoded in the k -th hop for $k = \{1, \dots, m-1\}$. Hence, the overall outage probability of the M -hop relay network is expressed as

$$\mathcal{P}_{M,L}(\mathbf{R}, \mathbf{P}) = \sum_{m=1}^M \prod_{k=1}^{m-1} (1 - p_{k,L_k}(R_k, P_k)) p_{m,L_m}(R_m, P_m),$$

where $\mathbf{R} = \{R_1, \dots, R_M\}$ and $\mathbf{P} = \{P_1, \dots, P_M\}$. For a given R_1, P_1 , and L , the average number of transmissions in the first hop is given by

$$\begin{aligned} \mathbb{E}[S_1|R_1, P_1; L] &= \sum_{k=1}^{L_1-1} k \cdot q_{1,k}(R_1, P_1) + L_1 p_{1,L_1-1}(R_1, P_1) \\ &= \sum_{k=0}^{L_1-1} p_{1,k}(R_1, P_1). \end{aligned} \quad (2)$$

¹This condition may not be needed for theoretical analysis, but it provides the practical aspect of HARQ techniques such as the coding complexity or buffer-size limitation.

The average number of transmissions in the m -th ($2 \leq m \leq M$) hop is given by

$$\begin{aligned} \mathbb{E}[S_m|R_1, \dots, R_m, P_1, \dots, P_m; L] \\ = \prod_{k=1}^{m-1} (1 - p_{k,L_k}(R_k, P_k)) \sum_{l=0}^{L_m-1} p_{m,l}(R_m, P_m). \end{aligned} \quad (3)$$

The optimization problem under delay, power, and outage-probability constraints is formulated as:

$$\max_{\mathbf{R}, \mathbf{P}} \mathcal{T}(\mathbf{R}, \mathbf{P}; L) \quad (4a)$$

$$\text{subject to } \sum_{m=1}^M P_m \leq P^{\text{total}}, \quad (4b)$$

$$\mathcal{P}_{M,L}(\mathbf{R}, \mathbf{P}) \leq \epsilon, \quad (4c)$$

$$R_m \geq 0, P_m \geq 0 \text{ for all } m, \quad (4d)$$

where $\mathcal{T}(\mathbf{R}, \mathbf{P}; L) = 1/(\sum_{m=1}^M \mathbb{E}[S_m|L]/R_m)$ and $\epsilon > 0$.

B. Gaussian Approximation

Unfortunately, problem (4) is mathematically intractable since $\mathcal{P}_{M,L}(\mathbf{R})$ and $\mathbb{E}[S_m|R_1, \dots, R_m, P_1, \dots, P_m; L]$ for $1 \leq m \leq M$ are composed of $p_{m,k}(R_m, P_m)$ whose output is hard to be obtained. Therefore, we use the Gaussian approximation [12] proposed for computing mutual information of the HARQ-IR scheme in the single-hop, which is given by

$$\sum_{l=1}^k \log_2(1 + |h_{m,l}|^2 P_m) \rightarrow \mathcal{N}(k\mu_m(P_m), k\gamma_m^2(P_m)),$$

where

$$\mu_m(P_m) = \mathbb{E}[\log_2(1 + |h_l|^2 P_m)] = \frac{e^{\frac{1}{(\sigma_m^2 P_m)}}}{\ln 2} E_1\left(\frac{1}{(\sigma_m^2 P_m)}\right),$$

$$\gamma_m^2(P_m) = \frac{2e^{\frac{1}{(\sigma_m^2 P_m)}}}{\sigma_m^2 P_m (\ln 2)^2} G_{3,4}^{4,0}\left(\frac{1}{\sigma_m^2 P_m} \middle| \begin{matrix} 0,0,0 \\ 0,-1,-1,-1 \end{matrix} \right) - \mu_m(P_m)^2,$$

where $E_1(x)$ indicates the exponential integral and $G_{p,q}^{m,n}(x | \begin{matrix} a_1, \dots, a_p \\ b_1, \dots, b_q \end{matrix})$ indicates the Meijer G-function. Then, the approximated version of $p_{m,k}(R_m, P_m)$ is defined as

$$\hat{p}_{m,k}(R_m, P_m) = Q\left(\frac{k\mu_m(P_m) - R_m}{\sqrt{k}\gamma_m(P_m)}\right). \quad (5)$$

In algorithms of this letter, we use (5) instead of $p_{m,k}(R_m, P_m)$ unless otherwise mentioned.

C. Proposed Algorithm

Although $p_{m,k}(R_m, P_m)$ is approximated, problem (4) is still not tractable, so that we propose a sub-optimal, two-step approach jointly optimizing \mathbf{R} and \mathbf{P} : 1) We first try to optimize \mathbf{R} for a given \mathbf{P} , i.e., $\mathbf{R}(\mathbf{P})$ is found. 2) We optimize \mathbf{P} by substituting $\mathbf{R}(\mathbf{P})$.

In the first step, for a given \mathbf{P} , problem (4) is rewritten as:

$$\min_{\mathbf{R}} U(\mathbf{R}, \mathbf{P}; L), \quad \text{s.t. } \hat{\mathcal{P}}_{M,L}(\mathbf{R}, \mathbf{P}) \leq \epsilon, R_m > 0, \quad (6)$$

where $U(\mathbf{R}, \mathbf{P}; L) = \sum_{m=1}^M ((\prod_{k=1}^{m-1} (1 - \hat{p}_{k,L_k}(R_k, P_k))) \sum_{l=0}^{L_m-1} \hat{p}_{m,l}(R_m, P_m)) / R_m$ and $\hat{\mathcal{P}}_{M,L}(\mathbf{R}, \mathbf{P}) = \sum_{m=1}^M \prod_{k=1}^{m-1} (1 - \hat{p}_{k,L_k}(R_k, P_k)) \hat{p}_{m,L_m}(R_m, P_m)$. However, problem (6) is

still difficult because $U(\mathbf{R}, \mathbf{P}; L)$ has multiple local minimum points on the M -dimensional space. Hence, we propose a sub-optimal solution of problem (6) by setting additional constraints: $\hat{p}_m(R_m, P_m) = \epsilon_m, \forall m$ and $\sum_{m=1}^M \epsilon_m = \epsilon$ (We discuss the selection of ϵ_m later). These two constraints always satisfy $\hat{P}_{M,L}(\mathbf{R}, \mathbf{P}) \leq \epsilon$. R_m satisfying $\hat{p}_m(R_m, P_m) = \epsilon_m$ is given by

$$R_{\text{sub},m}(P_m) = \left[L_m \mu_m(P_m) - \sqrt{L_m} \gamma_m(P_m) Q^{-1}(\epsilon_m) \right]^+, \quad (7)$$

where $(\cdot)^+$ denotes $\max(0, \cdot)$. Actually, $R_{\text{sub},m}(P_m)$ becomes the maximum R_m subject to $\hat{p}_m(R_m, P_m) \leq \epsilon_m$ since $\hat{p}_{m,L_m}(R_m, P_m)$ is an increasing function of R_m .

In the second step, we substitute $\mathbf{R}_{\text{sub}}(\mathbf{P})$ to $U(\mathbf{R}, \mathbf{P}; L)$. Since $U(\mathbf{R}_{\text{sub}}(\mathbf{P}), \mathbf{P}; L)$ is also intractable to optimize it, we approximate $U(\mathbf{R}_{\text{sub}}, \mathbf{P}; L)$ using the following theorem:

Theorem 1: $U(\mathbf{R}_{\text{sub}}, \mathbf{P}; L)$ converges to the following function as $L \rightarrow \infty$:

$$\lim_{L \rightarrow \infty} U(\mathbf{R}_{\text{sub}}, \mathbf{P}; L) = \sum_{m=1}^M \frac{\prod_{k=1}^{m-1} (1 - \epsilon_k)}{\mu_m(P_m)}. \quad (8)$$

Proof: See Appendix A. \blacksquare

From (8), we can observe that the selection of ϵ_m has a marginal effect if ϵ has sufficiently small values ($\epsilon \leq 0.01$) since $\prod_{k=1}^{m-1} (1 - \epsilon_k) \approx 1$. In numerical examples, we use $\epsilon_m = \epsilon/M, \forall m$. For given $\{\epsilon_1, \dots, \epsilon_M\}$ such that $\sum_{m=1}^M \epsilon_m = \epsilon$, problem (4) is approximated as

$$\min_{\mathbf{P}} \bar{U}(\mathbf{P}) \text{ s.t. } \sum_m P_m \leq P^{\text{total}}, P_m \geq 0 \quad \forall m, \quad (9)$$

where $\bar{U}(\mathbf{P}) = \sum_{m=1}^M ((\prod_{k=1}^{m-1} (1 - \epsilon_k)) / (\mu_m(P_m)))$. We solve (9) using Karush-Khun-Tucker (KKT) conditions written as

$$\frac{d\bar{U}(\mathbf{P})}{dP_m} - \lambda_m = -\lambda_0, \quad \forall m \quad (10a)$$

$$\lambda_m P_m = 0, \text{ and } \lambda_m \geq 0, \quad \forall m \quad (10b)$$

$$\sum_{m=1}^M P_m \leq P^{\text{total}}, \text{ and } P_m \geq 0, \quad \forall m. \quad (10c)$$

If P_m becomes zero, $\bar{U}(\mathbf{P})$ becomes infinity. Thus, the case of $P_m = 0$ is neglected and we assume that $\lambda_m = 0, \forall m$. Moreover, the optimal \mathbf{P} is on the boundary of $\sum_{m=1}^M P_m = P^{\text{total}}$. Then, we can find the solutions satisfying the following two constraints: $d\bar{U}(\mathbf{P})/dP_m = -\lambda_0, \forall m$, and $\sum_{m=1}^M P_m = P^{\text{total}}$. For convenience, we denote $V_m(P_m) = d\bar{U}(\mathbf{P})/dP_m$, expressed by

$$V_m(x) = \frac{\ln 2 e^{-\frac{1}{\sigma_m^2 x}} \prod_{k=1}^{m-1} (1 - \epsilon_k)}{x E_1\left(\frac{1}{\sigma_m^2 x}\right)} \left(\frac{1}{\sigma_m^2 x} - \frac{e^{-\frac{1}{\sigma_m^2 x}}}{E_1\left(\frac{1}{\sigma_m^2 x}\right)} \right).$$

Let $V_m^{-1}(a) = x$ be the inverse function of $V_m(x) = a$. Unfortunately, $V_m^{-1}(a)$ has no closed-form and the value of $V_m^{-1}(a)$ can be obtained by numerical search. Now, we find the optimal λ_0 satisfying $\sum_{m=1}^M V_m^{-1}(-\lambda_0) = P^{\text{total}}$. Since $V_m^{-1}(a)$ can be empirically regarded as a monotonic increasing function as discussed in Appendix B, the optimal λ_0 can be found using the bisection method, so that \mathbf{P}^* and $\mathbf{R}_{\text{sub}}(\mathbf{P}^*)$ are obtained. The

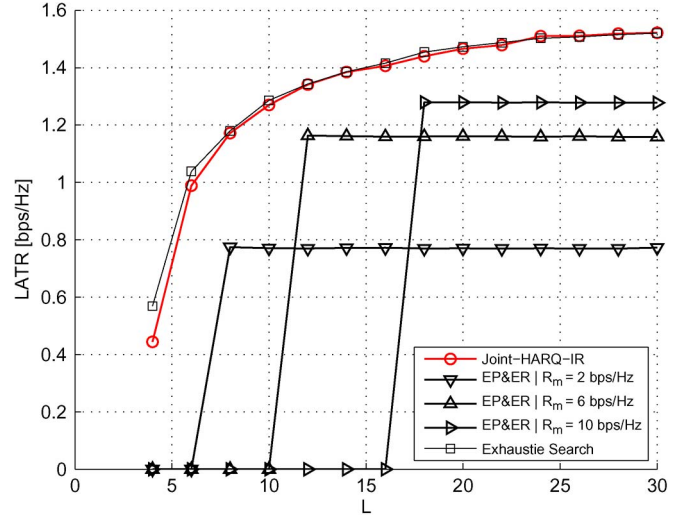


Fig. 1. LATR of the proposed algorithm according to L for $M = 2$, $P^{\text{total}}/\Omega = 10$ [dB] and $\epsilon = 0.01$ when $(\sigma_1^2, \sigma_2^2) = (1, 10)$.

complexity of the proposed algorithm increases linearly as M increases. Meanwhile, the complexity of the exhaustive search for finding the optimal solution in (4) increases on the order of $2M$ as M increases.

IV. NUMERICAL RESULTS

We use Monte Carlo simulation to obtain real LATRs of each scheme. We assign zero LATR when each scheme does not satisfy $\hat{P}_{M,L}(\mathbf{R}, \mathbf{P}) \leq \epsilon$. Fig. 1 shows the LATR versus L for $M = 2$, $P^{\text{total}}/\Omega = 10$ [dB] and $\epsilon = 0.01$ when $(\sigma_1^2, \sigma_2^2) = (1, 10)$. In Fig. 1, the conventional link adaptation technique which operates with an equal power and equal rate over the network is compared with the proposed algorithm, of which transmission rates are varied ($R_m = 2, 6, 10$ b/s/Hz, $\forall m$) and transmit power is fixed ($P_m = P^{\text{total}}/M, \forall m$). The proposed algorithm significantly outperforms the conventional link adaptation technique irrespective of L . LATR of the proposed algorithm increases as L increases, but LATR of the conventional link adaptation technique with equal power and equal rate is saturated as L increases. The smallest LATR gap between the proposed algorithm and the conventional technique with $R_m = 10$ bps/Hz is approximately 0.16 b/s/Hz at $L = 18$, which is about 13% LATR gain. Note that the proposed algorithm yields very similar LATR performance to the exhaustive search algorithm finding the optimal solution of problem (4) with Gaussian approximation.

Fig. 2 shows the LATR of the proposed algorithm according to P^{total}/Ω for $M = 2$, $L = 10$, and $\epsilon = 0.01$ when $(\sigma_1^2, \sigma_2^2) = (1, 10)$. In Fig. 2, we also consider the joint power and rate optimization technique as a conventional scheme, which is proposed for the same network with Chase-combining HARQ in [10]. The proposed algorithm outperforms all the conventional techniques in terms of LATR and the performance gaps between the proposed algorithm and the conventional techniques increase as the available power (or equivalently SNR) increases. For example, the proposed algorithm achieves about 29% LATR gain when $P^{\text{total}}/\Omega = 18$ dB, compared with the optimal link adaptation technique with HARQ-CC [10]. We can also observe that the proposed algorithm yields very similar LATR performance to the exhaustive search algorithm. Since

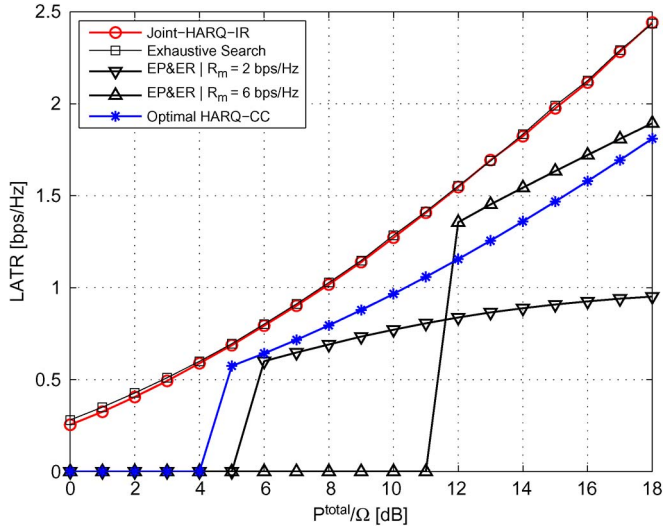


Fig. 2. LATR of the proposed algorithm according to P^{total}/Ω for $M = 2$, $L = 10$, and $\epsilon = 0.01$ when $(\sigma_1^2, \sigma_2^2) = (1, 10)$.

the proposed scheme uses the Gaussian approximation, it does not strictly guarantee $\mathcal{P}_{M,L}(\mathbf{R}, \mathbf{P}) \leq \epsilon$. For Figs. 1 and 2, the maximum outage probability of the proposed scheme is 0.019.

V. CONCLUSION

In this letter, we investigate a problem of adjusting transmission rates and powers of all transmitting nodes in order to maximize the LATR in a multihop relay network for given constraints on sum of the transmission powers and the total number of time slots for transmitting a single packet. We utilize an approximation method in order to propose a feasible and effective solution since the original optimization problem is not mathematically tractable. It is shown that the proposed algorithm significantly outperforms the conventional techniques through extensive simulations.

APPENDIX A PROOF OF THEOREM 1

It is sure that $\hat{p}_{k,L_k}(R_{\text{sub},k}, P_k) = \epsilon_k$. Then, Theorem 1 is proved if we prove $(R_{\text{sub},m}/(\sum_{l=0}^{L_m-1} \hat{p}_{m,l}(R_{\text{sub},m}, P_m))) \rightarrow \mu(P_m)$ as $L \rightarrow \infty$. Theorem 2 of [12] considered this convergence without the Gaussian approximation. We modify the proof of Theorem 2 of [12] for the Gaussian approximation case here. Because $L_m \rightarrow \infty$ as $L \rightarrow \infty$ ($\because L_m = L/M$), we observe $\lim_{L_m \rightarrow \infty} [(R_{\text{sub},m}/L_m) \cdot (L_m/(\sum_{l=0}^{L_m-1} \hat{p}_{m,l} \times (R_{\text{sub},m}, P_m)))]$. We obtain $\lim_{L_m \rightarrow \infty} (R_{\text{sub},m}/L_m) = \mu_m(P_m)$ and $(L_m/(\sum_{l=0}^{L_m-1} \hat{p}_{m,l}(R_{\text{sub},m}, P_m))) \geq 1$. In addition, we define Gaussian random variables $Y_l \sim \mathcal{N}(l\mu_m(P_m), l\gamma_m^2(P_m))$. Then, $\sum_{l=0}^{L_m-1} \hat{p}_{m,l}(R_{\text{sub},m}, P_m)$ is lower-bounded by

$$\begin{aligned} & \sum_{l=0}^{L_m-1} \hat{p}_{m,l}(R_{\text{sub},m}, P_m) \\ &= 1 + \sum_{l=1}^{L_m-1} \Pr \left[Y_l \leq L_m \mu_m - \sqrt{L_m} \gamma_m Q^{-1}(\epsilon_m) \right] \\ &\geq 1 + \beta \Pr \left[Y_\beta \leq L_m \mu_m - \sqrt{L_m} \gamma_m Q^{-1}(\epsilon_m) \right] \quad (\text{A.1}) \end{aligned}$$

where $\beta = \lceil L_m - L_m^{3/4} \rceil$. Since $(Y_\beta/\beta) \sim \mathcal{N}(\mu_m, (\gamma_m/\beta))$, we have

$$\lim_{L_m \rightarrow \infty} \Pr \left[Y_\beta \leq L_m \mu_m - \sqrt{L_m} \gamma_m Q^{-1}(\epsilon_m) \right] = 1.$$

Therefore, $\lim_{L \rightarrow \infty} ((\sum_{l=0}^{L_m-1} \hat{p}_{m,l}(R_{\text{sub},m}, P_m))/L_m) \geq 1$. Finally, we obtain $\lim_{L_m \rightarrow \infty} [(R_{\text{sub},m}/L_m) \cdot (L_m/(\sum_{l=0}^{L_m-1} \hat{p}_{m,l}(R_{\text{sub},m}, P_m)))] = \mu_m(P_m)$.

APPENDIX B

DISCUSSION OF MONOTONIC PROPERTY OF $V_m^{-1}(a)$

If each summand of $\bar{U}(\mathbf{P})$, $(\ln 2 \prod_{k=1}^{m-1} (1 - \epsilon_k)) / (\exp(1/(\sigma_m^2 P_m)) E_1(1/(\sigma_m^2 P_m)))$, is a strictly convex function, $V_m(P_m)$ becomes monotonically increasing function. However, it is hard to find the convexity of $(\ln 2) / (\exp(1/(\sigma_m^2 P_m)) E_1(1/(\sigma_m^2 P_m)))$ due to its complicated form. Instead, we prove the convexity of its upper and lower bounds:

$$\begin{aligned} \frac{\prod_{k=1}^{m-1} (1 - \epsilon_k)}{\log_2(1 + \sigma_m^2 P_m)} &\leq \frac{\ln 2 \prod_{k=1}^{m-1} (1 - \epsilon_k)}{\exp\left(\frac{1}{\sigma_m^2 P_m}\right) E_1\left(\frac{1}{\sigma_m^2 P_m}\right)} \\ &\leq \frac{2 \prod_{k=1}^{m-1} (1 - \epsilon_k)}{\log_2(1 + 2\sigma_m^2 P_m)}, \quad (\text{B.1}) \end{aligned}$$

where we used the inequality $(1/2)e^{-x} \ln(1 + (2/x)) \leq E_1(x) \leq e^{-x} \ln(1 + (1/x))$ and $1/(\ln(1 + x))$ is a convex function of x . Numerical observation of (B.1), omitted in this letter, shows that each summand of $\bar{U}(\mathbf{P})$ is tightly bounded by its upper and lower bounds, so that $V_m(P_m)$ and $V^{-1}(a)$ are empirically assumed to be a monotonically increasing function.

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